LINEAR INTERPOLATION

SUMMIT TRICKS AND TRAPS

SERIES - 1 / ARTICLE - 9

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By **GreenPoint Summit Team**

GreenPoint> Summit

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1. Linear Interpolation Introduction

Linear interpolation is a method to construct new data points with existing ones. Since antiquity, gaps in tables have reportedly been filled via linear interpolation, frequently using astronomical data. These days, math and finance both employ this technique. The estimation of any desired value at a particular known coordinate point benefits greatly from linear interpolation. It is a method of curve fitting that uses linear polynomials to plot new data points, which lie in the range of a discrete set of known data points.

1.1. Linear Interpolation in Math

In mathematics, linear interpolation is a curve fitting method that uses linear polynomials to construct new data points within the range of a discrete set of known ones. Because linear interpolation is helpful while searching for a value between a given set of points, in math, it is often used as a strategy that implies using a straight line to connect the given set of points on the positive and negative side of the unknown point.

1.1.1. Formulas

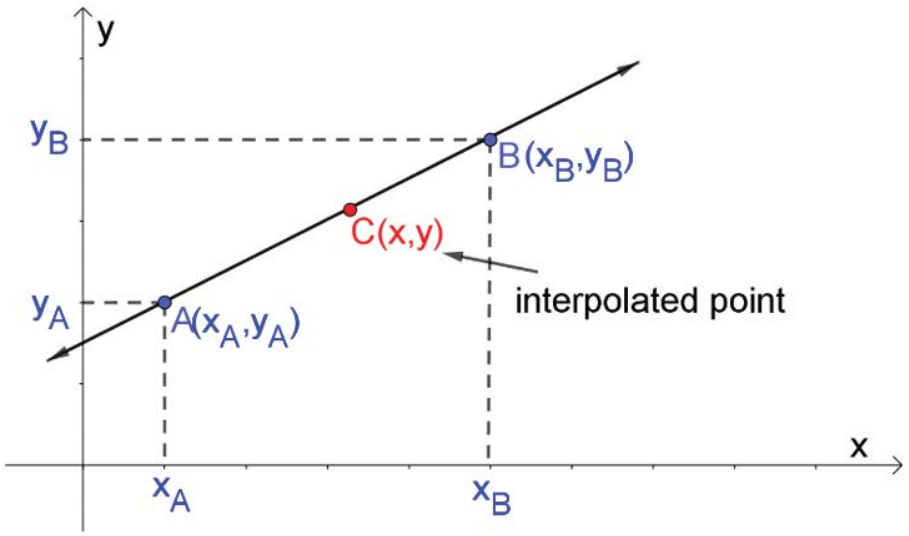
If the two known data points are (a, b) and (c, d) and the linear interpolant (x, y) will be the straight line between (a, b) and (c, d). Let us suppose a<c, then for x in the interval a<x<c, the value y is given from the equation of slopes:

$$\frac{y-b}{x-a} = \frac{d-b}{c-a}$$

Which can be derived into:

$$y = b * \frac{c-x}{c-a} + d * \frac{x-a}{c-a}$$

For linear interpolation on a set of data points $(x0, y0)$, $(x1, y1)$, ..., (xn, yn) is defined as the concatenation of linear interpolants between each pair of data points. This results in a continuous curve, with a discontinuous derivative (in general), thus of differentiability class C^0.



1.2. Linear Interpolation in Summit

In finance, the linear interpolation method is frequently used to generate data points and for curve fitting. Investors frequently use the Linear Interpolation approach using data from other points to estimate the value for a point where there is no data.

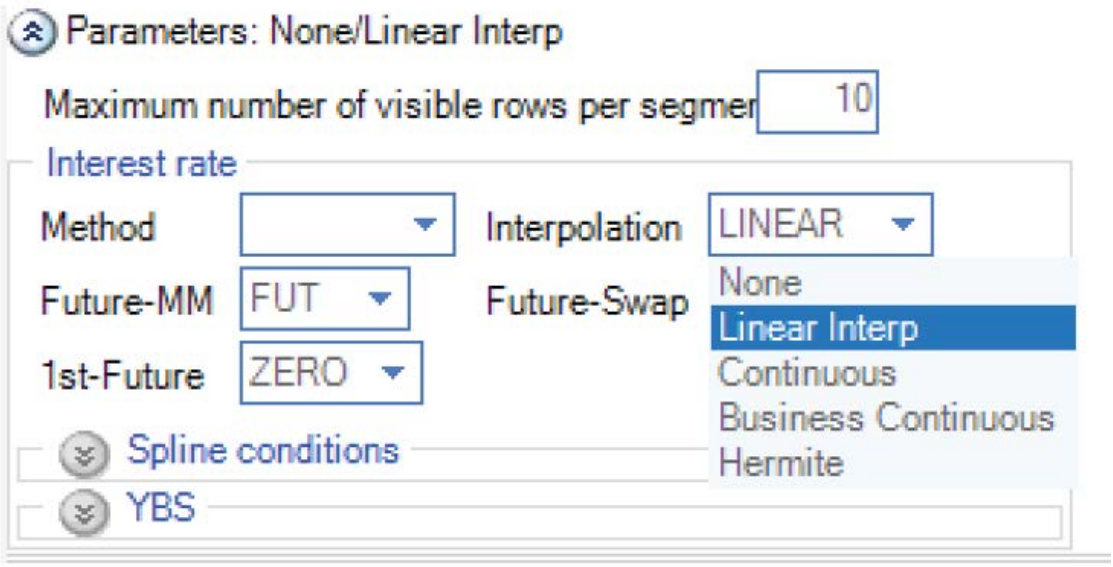
In Summit, market data calculations and curve construction use the linear interpolation method. When determining unknown values between existing nodes for term and strike, the system uses Linear Interpolation logic by default.

The user can specify the interpolation method used on zero rates by selecting Linear Interpolated in the Interpolation method Parameter fields of the Interest Rate Market Data Setup.

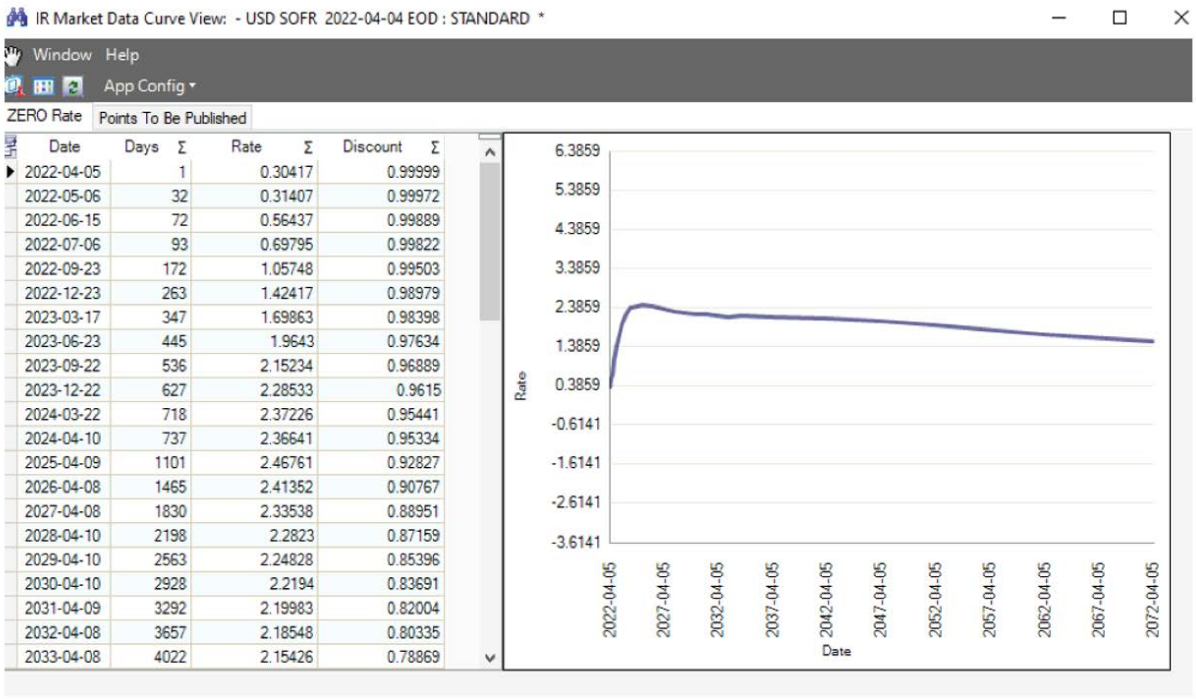
The options are the following:

- None
- Linear Interpolated
- Continuous
- Business Continuous

Summit: Interest Rate Market Data Application



Summit: Example for USD SOFR zero rate curve using Linear Interpolation



Date	Days	Rate	Discount
2022-04-05	1	0.30417	0.99999
2022-05-06	32	0.31407	0.99972
2022-06-15	72	0.56437	0.99889
2022-07-06	93	0.69795	0.99822
2022-09-23	172	1.05748	0.99503
2022-12-23	263	1.42417	0.98979
2023-03-17	347	1.69863	0.98398
2023-06-23	445	1.9643	0.97634
2023-09-22	536	2.15234	0.96889
2023-12-22	627	2.28533	0.9615
2024-03-22	718	2.37226	0.95441
2024-04-10	737	2.36641	0.95334
2025-04-09	1101	2.46761	0.92827
2026-04-08	1465	2.41352	0.90767
2027-04-08	1830	2.33538	0.88951
2028-04-10	2198	2.2823	0.87159
2029-04-10	2563	2.24828	0.85396
2030-04-10	2928	2.2194	0.83691
2031-04-09	3292	2.19983	0.82004
2032-04-08	3657	2.18548	0.80335
2033-04-08	4022	2.15426	0.78869

1.2.1. Formulas used in Summit

Summit Zero-coupon rate Calculation

With two known sets of zero-coupon rate and time, (Z_a, t_a), (Z_b, t_b), the zero-coupon rate of time c is:

$$Z_c = \frac{Z_a * (t_b - t_c) + Z_b * (t_c - t_a)}{t_b - t_a}$$

Exponential Form

Using exponential form of the discount curve, the preceding equation can be expressed using the discount factor on points A and B.
Solving the previous equation for a time interval, y, results in:

$$y = \frac{t_c - t_a}{t_b - t_a}$$

Summit Discount Factors Calculation

By using the above two equations, we can get a discount factor for point c:

$$DF_c = \exp\left(-t_c * \left((1-y) * Z_a + y * Z_b\right)\right)$$

Which can be derived to:

$$DF_c = DF_a^{\frac{t_c}{t_a} * \frac{t_b - t_c}{t_b - t_a}} * DF_b^{\frac{t_c}{t_b} * \frac{t_c - t_a}{t_b - t_a}}$$

1.2.2. Conclusion

When building curves in Summit, various interpolation methods will be selected based on the conditions and market data. The advantage of the linear interpolation method is that it is very reliable, easy to implement, and is usually the starting point for developing yield curve models. The drawback of this method is that it does not guard against negative forward rates and may have jumps between forward rates, thus impairing the continuity of the curve. Therefore, users should evaluate and compare the stability and continuity of curves when selecting interpolation methods for curves by employing various interpolation techniques.

GreenPoint> Summit

ABOUT GREENPOINT SUMMIT

- GreenPoint Summit is a comprehensive platform encompassing new implementations, version and module upgrades, product and application development, test automation, cloud migration, and system maintenance
- Our quantitative services and platforms include Libor Replacement Simulation Tool (LRST), curve creation, recreation and management, model validation and documentation, and creation of challenger models for regulatory compliance.
- Our summit professionals also provide data porting, migration and management as well as cloud services.
- Over the last year we have completed several projects including full system upgrades, Libor/RFR migration, replacement of valuation frameworks, and custom code creation and testing for large global banks and insurers.

GreenPoint> Financial

ABOUT GREENPOINT FINANCIAL

- GreenPoint Financial is a division of GreenPoint Global, which provides software-enabled services, content, process and technology services, to financial institutions and related industry segments.
- GreenPoint is partnering with Finastra across multiple technology and services platforms.
- Founded in 2006, GreenPoint has grown to over 500 employees with a global footprint. Our production and management teams are in the US, India, and Israel with access to subject matter experts.
- GreenPoint has a stable client base that ranges from small and medium-sized organizations to Fortune 1000 companies worldwide. We serve our clients through our deep resource pool of subject matter experts and process specialists across several domains.
- As an ISO certified company by TÜV Nord, GreenPoint rigorously complies with ISO 9001:2015, ISO 27001:2013, and ISO 27701:2019 standards.

Sanjay Sharma, PhD

FOUNDER AND CHAIRMAN

Sanjay provides strategic and tactical guidance to GreenPoint senior management and serves as client ombudsman. His career in the financial services industry spans three decades during which he has held investment banking and C-level risk management positions at Royal Bank of Canada (RBC) Goldman Sachs, Merrill Lynch, Citigroup, Moody's, and Natixis.

Sanjay is the author of "Risk Transparency" (Risk Books, 2013), Data Privacy and GDPR Handbook (Wiley, 2019), and co-author of "The Fundamental Review of Trading Book (or FRTB) - Impact and Implementation" (Risk Books, 2018).

Sanjay was the Founding Director of the RBC/Hass Fellowship Program at the University of California at Berkeley and has served as an advisor and a member of the Board of Directors of UPS Capital (a Division of UPS). He has also served on the Global Board of Directors for Professional Risk International Association (PRMIA).

Sanjay holds a PhD in Finance and International Business from New York University and an MBA from the Wharton School of Business and has undergraduate degrees in Physics and Marine Engineering. As well as being a regular speaker at conferences, Sanjay actively teaches postgraduate level courses in business and quantitative finance at EDHEC (NICE, France), Fordham, and Columbia Universities.

Marcus Cree

MANAGING DIRECTOR AND HEAD OF FINANCIAL PRODUCTS AND SERVICES

Marcus heads GreenPoint Financial Technology and Services and has conceptualized and directed design and management of its Risk Management and Treasury and Capital Markets platforms. These areas encompass lending (including CECL), sustainable finance, and LIBOR/RFR transition for enterprise systems.

Marcus has over two decades of experience in Risk Management, working on both the buy and sell sides of the financial services industry. He has worked on capital markets and lending risk technology and management projects in over 50 countries and brings a unique perspective on the nuances and differences across regulatory regimes around the world.

Before joining GreenPoint, Marcus was a member of the strategic leadership team for risk management at Finastra for several years. Prior to Finastra, Marcus was the US Head of Risk Solutions for FIS. He started his professional career at Deutsche Bank in London as a Quantitative Analyst.

Marcus is a prolific conference speaker, thought leader, and author in risk management spanning market, credit, and liquidity risks. He also publishes papers on sustainability and green finance regularly.

Marcus graduated from Leicester University in the UK, after studying Pure Mathematics, Phycology, and Astronomy. Since graduation, Marcus has continually gained risk-specific qualifications including the FRM (GARP's Financial Risk Manager) and the SCR (GARP's Sustainability and Climate Risk). Marcus's latest academic initiative is creating and teaching a course on Green Finance and Risk Management at New York University's Tandon School of Engineering.

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